Two Term Approximation for Near Field Analysis of Wire Scatterers

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ABSTRACT

This paper presents an analysis of near field of wire scatterers. In this paper, the exact distribution on wire scatterers is approximated using a more accurate two term approximation for the ease of closed form analysis. It has been shown that the results obtained using the two term approximation are very close to the cosinusoidal distribution for the element length up to 0.7λ with less than 5% error in the field evaluated from this approximation. This analysis formulation is useful in estimating fields form wire scatterers, mutual coupling in an array of scatterers or analysis of Yagi-Uda antennas.

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1. INTRODUCTION

Wire scatterer is typically a simple metal rod acting as a passive radiator. Scatterer or parasitic element is a conductive element in a radio antenna which is not electrically connected to anything else. In case of Yagi-uda antenna [1], there is one driven element and others are scatterers (either directors or reflectors). Wire scatterers are also used in applications like wire grid polarizers [2], frequency selective surfaces, chaff, *etc.* Analysis of wire scatterers is also essential in estimating the effect on and degradation in pattern due to wire scatteres such as adjacent metallic structures, support rods, interference or pick up due to PCB traces etc. In case of scatterers used in an array, the fields are often measured at very close distance where the near field criteria are useful for the analysis. Mutual coupling between scatterers needs to be studied accurately in case of array of scatterers or multiple scatterers in close proximity.

The current distribution on dipoles is often approximated with piecewise sinusoidal (PWS) distribution which is of the form $\sin[k(l-|z|)]$, where $2l$ is the length of the scatterer and k is the wavenumber [3, 4]. Scatterers are also often analysed using PWS distribution similar to dipoles, but as the length of the scatterer deviates from half a wavelength, the PWS does not hold true. Here Cosine distribution similar to waveguide fed slot aperture distribution is more appropriate and should be used for more accurate annalysis. However, near fields from cosine distribution of the form cos ($\pi/2l\ z$) cannot be analyzed in closed form. It is this change from free space wavenumber to that where k is not involved that causes trouble in calculating the closed form expression in Cosine distribution [5]. Therefore, here the near field analysis is done numerically using two term approximation distribution since it is very close to the cosine distribution and it can also be analyzed in closed form [6, 7]. This paper highlights the applicability of two term approximation to wire scatterers of different lengths. The maximum length of scatterer for which near fields may be evaluated in closed form using the proposed two term approximation with an error of less than 5% is also found out. The analysis methodology is presented in Section 2 and results are discussed in Section 3.

2. METHODOLOGY

Figure 1 shows the geometry of the wire scatterer with the co-ordinate system. The scatterer is assumed to be of length $2l$, diameter d and placed along z-axis in the yz plane. The diameter of the scatterer is much less than the length of the scatterer. The scatterer can be replaced by an equivalent linear electric current source along the axis of the scatterer or the z-axis in this case. The scatterer has a field pattern with azimuthal symmetry similar to a dipole. Thus, the field point can be conveniently located in the yz plane ($\Phi = \frac{\pi}{2}$), without loss of generality. The centre of the scatterer is at origin O (0, 0, 0) and the observation point P is at (0, y_0, z_0). P= (y_0,$\frac{\pi}{2}$, z_0) in cylindrical coordinates and P = (R_0,$\frac{\pi}{2} - \theta$, $\frac{\pi}{2}$) in spherical coordinates. The centre to the field distance is R_0. z' is any point on the scatterer along z axis. R is the distance between z' and the observation point P.

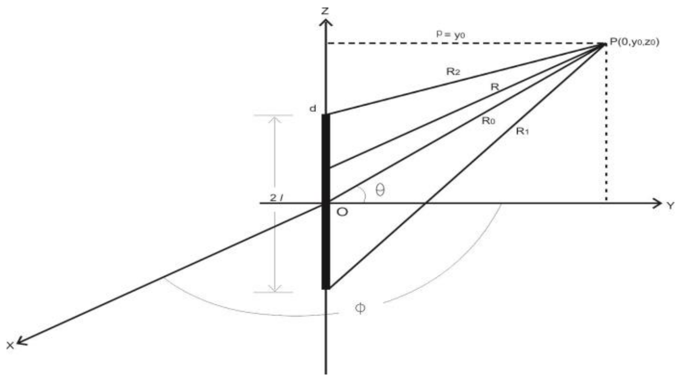


Figure 1. Geometry of the Problem

The total field at observation point P is,

$$\mathrm{Et} = \mathrm{Ez}\bar{z} + \mathrm{E}\rho\bar{\rho} \quad (1)$$

The electric current on the scatterer is assumed cosinusoidal or J= cos ($\frac{\pi}{2l}$)$\bar{z}$.

Following the procedure in [3, 8],

$$E_z = \int_{-l}^{l} \frac{J\eta}{jk}\left(\frac{\partial^2 y}{\partial z^2} + k^2\right) G\ dz' \quad (2)$$

$$E_\rho = E_y = -\int_{-l}^{l} \frac{J\eta}{jk}\frac{\partial^2 G}{\partial z\, \partial y} dz' \quad (3)$$

where, $k = \frac{2\pi}{\lambda}$, λ is the free space wavelength, η = 120π is the free space impedance, $G = G(R) = \frac{e^{-jkR}}{4\pi R}$ is the free space Green's function and $R = \sqrt{(z - z')^2 + \rho^2}$.

2.1. Two Term Approximation

A two term approximation is similar to that in [6] is used for scatterer electric current distribution

$$J(z') = \cos\left(\frac{\pi}{2l} z'\right) \approx a_1\{\sin[k(l - |z'|)] + a_0 k(l - |z'|)\cos[k(l - |z'|)]\} \quad (4)$$

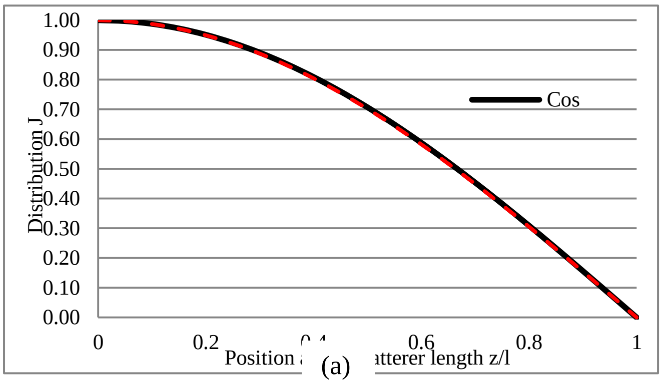
Here we approximate the current to a cosinusoid and the constants a_1 , a_0 are evaluated subject to

$$J(0) = 1 \text{ and } \frac{\partial J}{\partial z'} \; at \; z' = 0 \; is \; zero.$$

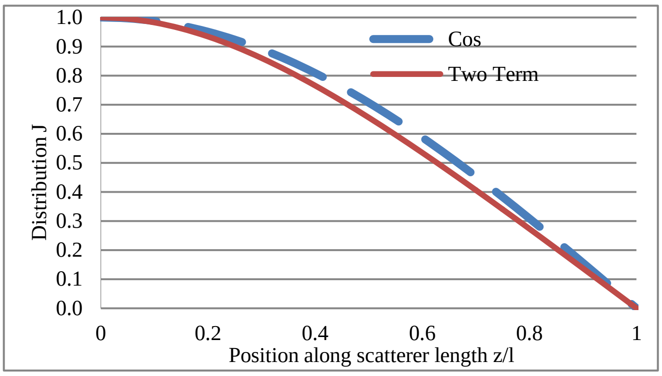
Then,

$$a_0 = \frac{1}{kl\,\tan(kl) - 1} \quad (5)$$

$$a_1 = \frac{1}{\sin(kl) + a_0 kl\,\cos(kl)} \quad (6)$$



(a)



(b)

Figure 2. Distribution for Cosinusoidal and two term approximation for scatterer lengths (a)0.6λ and (b)0.75λ

The scatterer distribution using cosine and two term approximation distribution is given in figure 2 for scatterer length *2l* = 0.6λ and *2l* = 0.75λ. It can be seen from graph that the two distributions are close to each other.

2.2. Near Field of Scatterers using Two Term Approximation

Using the two term approximation equation (4) for scatterer electric current distribution J in equations (2, 3), the equations for E_z and E_ρ can be written as,

$$E_z = \int_{-l}^{l} \frac{J(z')\eta}{2\pi jk} \left\{ 3(z-z')^2 \frac{e^{-jkR}}{R^5} + 3jk(z-z')^2 \frac{e^{-jkR}}{R^4} - [k^2(z-z')^2 + 1] \frac{e^{-jkR}}{R^3} - jk \frac{e^{-jkR}}{R^2} + k^2 \frac{e^{-jkR}}{R} \right\} dz' \quad (7)$$

$$E_{\rho} = \frac{-J(z\prime)\eta e^{-jkR}}{jk}\left[\frac{jky}{R}\left(\frac{1}{R^3}+\frac{jk}{R^2}\right)+\frac{3y}{R^5}+\frac{2jky}{R^4}\right]\frac{(z-z\prime)}{2\pi} \quad (8)$$

Where,

$$J(z') = a_1\{\sin[k(l-|z'|)] + a_0 k(l-|z'|)\cos[k(l-|z'|)]\}$$

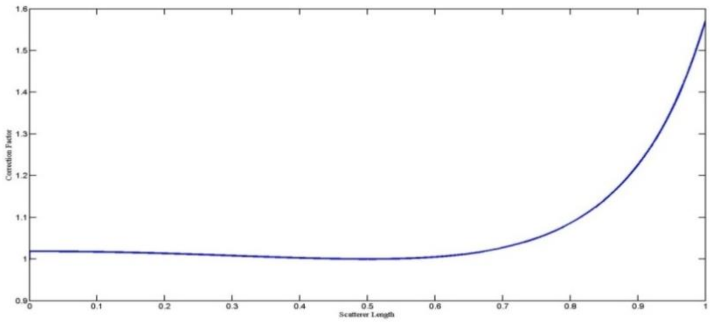


Figure 3. Correction factor for two term approximation

The fields given above can be further corrected for the difference in the two distributions by using a multiplicative cinstant γ [8].

$$\gamma = \frac{2kl}{\pi\{a_1(1-a_0)[1-\cos(kl)]+a_1 a_0 kl\sin(kl)\}} \quad (9)$$

The correction factor γ for two termapproximation that is the ratio of the first moments of the two distributions is plotted in figure 3.

The value of correction factor is nearly equal to one for scatterers length upto 0.5λ, however, it increases from unity above 0.5λ.

3. RESULTS AND DISCUSION

The percentage error variation in electric field with distance R_0/λ is plotted in figure 4 to Figure 8. Here the percentage error variation is the error between two term distribution and cosinusoidal distribution. It is calculated by taking the percentage of the difference between the two term and the cosine distributions with reference to cosine distribution. From figure 4, we can say that for scatterer length of $2l$=0.3λ, the error remains less than 1% over the entire angular range $0^0 \leq \theta \leq 90^0$ in near field and in the far field the error becomes negligible i.e. almost equal to zero and it remains constant.

For scatterer length $2l = 0.65\lambda$, the percentage error in magnitude of E-field is plotted in figure 5. Here the error in near field is more as compared to error in the previous result of figure 4. In this graph, the error in E-field magnitude is less than 2% over the entire angular range.

Similarly, results are plotted for scatterer lengths of 0.7λ, 0.8λ and 1λ in figures 6, 7 and 8 respectively.

In figure 6, for $2l = 0.7\lambda$, the error increases up to 4% in near field and the error remains constant and less than 1.5% in the far field. In figure 7 and 8, the length of scatterer is more than 0.7λ, it can be seen that the error increases considerably in the near field and becomes much more than 5%.

In figure 9, the relative error in near E-field magnitude is plotted over $0^0 \leq \theta \leq 90^0$ for $2l = 0.7\lambda$ and R_0=0.5λ, R_0=0.65λ, R_0=0.7λ, R_0=1λ. Here the error is more for wider angles as compared to $\theta = 0^0$. For wider angles (*e.g.*, $\theta = 90^0$), the error can be seen to increase with distance in the near field, but remains constant in the far field as seen from figure below.

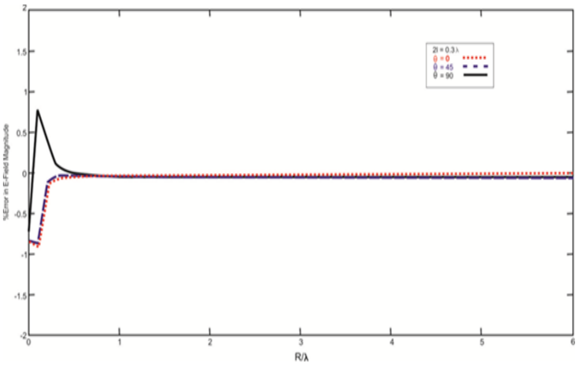


Figure 4. Variation of percentage error in magnitude of electric field calculated from Two Term Approximation with distance R0/λ for slot length 2l = 0.3λ at various angles θ.

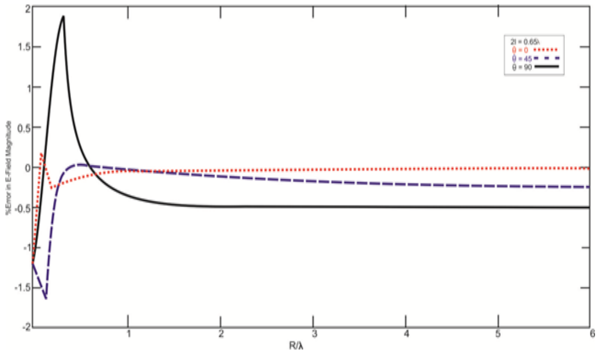


Figure 5. Variation of percentage error in magnitude of electric field calculated from Two Term Approximation with distance R_0/λ for slot length $2l = 0.65\lambda$ at various angles θ.

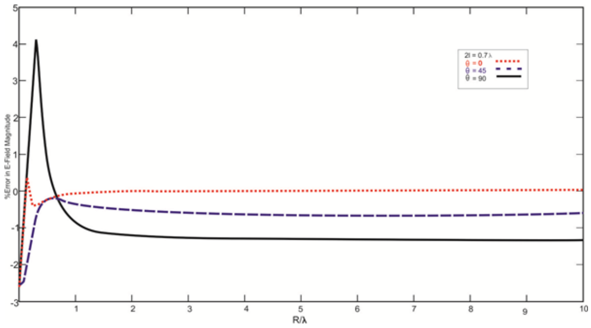


Figure 6. Variation of percentage error in magnitude of electric field calculated from Two Term Approximation with distance R_0/λ for slot length $2l = 0.7\lambda$ at various angles θ.

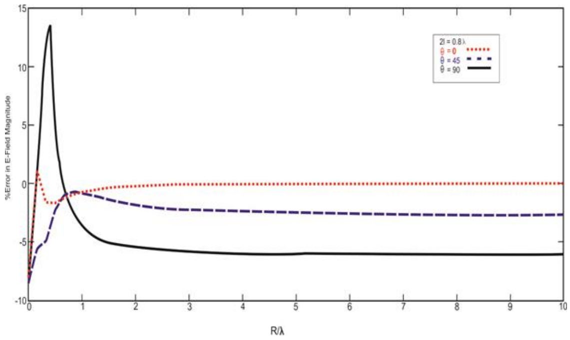


Figure 7. Variation of percentage error in magnitude of electric field calculated from Two Term Approximation with distance R_0/λ for slot length $2l = 0.8\lambda$ at various angles θ.

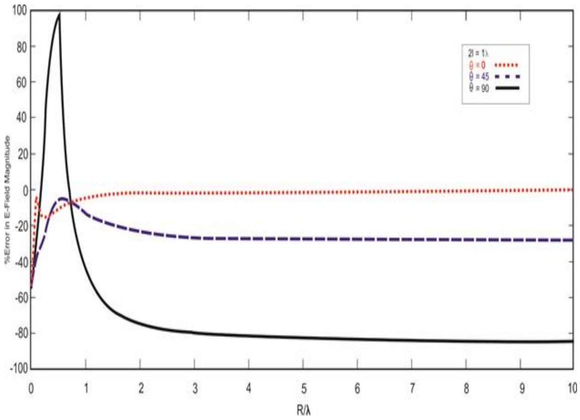


Figure 8. Variation of percentage error in magnitude of electric field calculated from Two Term Approximation with distance R_0/λ for slot length $2l = 1\lambda$ at various angles θ.

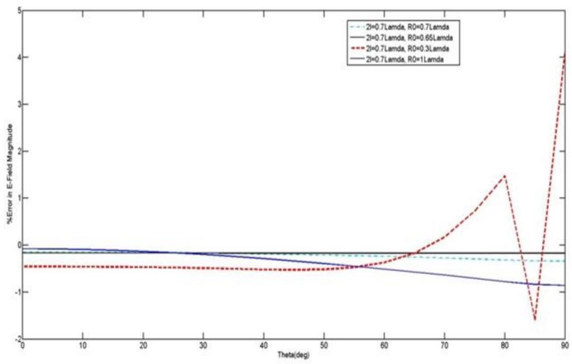


Figure 9. Variation of percentage error in magnitude of electric field calculated from Two Term Approximation with theta (θ) for scatterer length $2l = 0.7\lambda$ and $R_0 = 0.7\lambda$, 0.65λ, 0.3λ and 1λ.

The correction factor is taken into account while plotting the results. The numerical results are evaluated by dividing the scatterer into cells and employing three point Gaussian quadrature method over each cell. It is seen that the error remains less than 1% over the entire angular range $0^0 \leq \theta \leq 90^0$, for scatterer length $2l \leq 0.5\lambda$. The error increases for scatterer length $2l > 0.5\lambda$ where the approximate aperture distribution differs more significantly from cosinusoidal one.

The expressions derived in this paper are useful in calculating the closed form expressions using two term approximation distribution for near field of wire scatterers. Closed form expressions for slots are derived in [7]. Similar technique is used to find the closed form expressions for dipole scatterers. In this paper, results are calculated using the two term approximation and over different lengths of scatterers (upto length 1λ). This approximation can be used accurately upto 0.7λ length scatterers.

4. CONCLUSION

The two term approximation has been applied to cosinusoidal electric current on a wire scatterer. It has been shown that the two term approximation with correction factor is very close to cosinusoidal distribution as compared. The near fields from this approximate distribution have been compared with the exact fields for various lengths of scatterers. It has been shown that for scatterers with lengths $2l \leq 0.7\lambda$, the two term approximation developed is very close to the cosinusoidal distribution with less than 5% error. The approximation yields better than 1% error for scatterers up to lengths ≤ 0.5λ. For scatterers having lengths longer than 0.7λ, fields derived using this two term approximation distribution do not give satisfactory results and a better approximation is needed. The above formulation is very useful in estimating the near fields of scatterer numerically and can be used to evaluate the closed form expressions for near field with two term approximation.

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